

# Bridging the gap between causal inference and survival analysis: a censored edition

*STRATOS TG 7*

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# STRATOS

INITIATIVE

The logo for STRATOS INITIATIVE features the word "STRATOS" in a large, dark blue, sans-serif font. The letter "A" is stylized, with a green bar chart integrated into its shape. Below "STRATOS", the word "INITIATIVE" is written in a smaller, green, spaced-out, sans-serif font.

- STRengthening Analytical Thinking for Observational Studies
- Tremendous amount of development of new statistical methods
- In practice only few of these methods are used
- Big gap between methods development and methods applications
- Aim: provide guidance for statisticians and other data analysts

## STRATOS: 9 topic groups

- 1 Missing data
- 2 Selection of variables and functional forms in multivariable analysis
- 3 Initial data analysis
- 4 Measurement error and misclassification
- 5 Study design
- 6 Evaluating diagnostic tests and prediction models
- 7 Causal inference
- 8 Survival analysis
- 9 High-dimensional data

And several panels (in total 11 e.g. simulation panel)

More in Biometrics Bulletins and <https://stratos-initiative.org/>

## Some recent (2020) highlights of STRATOS

- TG 2: Overview of variable and Function Selection (Diagnostic and Prognostic Research 2020)
- TG 3: Review on reporting initial data analysis (BMC Medical Research Methodology 2020)
- TG 4: Two guidance papers on measurement error (Stat in Med 2020)
- Involved in SISAQOL, large European project on setting standards for analysis of Patients Reported Outcomes in Cancer Trials

MORE IN STRATOS MINI-SYMPOSIUM ON THURSDAY

## Topic group 7: causal inference

- courses
- didactic material (case studies)
- a simulation learner
- website. ([ofcaus.org](http://ofcaus.org))
- talks at meetings (+ organizing meetings)
- papers

Tutorial on causal questions and principled answers will appear shortly in Statistics in Medicine

## Key steps of causal inference

- 1 Define treatment with relevant levels/values corresponding to scientific question of study.
- 2 Define outcome.
- 3 Define the population(s) of interest.
- 4 Formalise potential outcomes
- 5 Specify the target causal effect ('Estimand')

## Key steps of causal inference

- 1 Define treatment with relevant levels/values corresponding to scientific question of study.
- 2 Define outcome.
- 3 Define the population(s) of interest.
- 4 Formalise potential outcomes
- 5 Specify the target causal effect ('Estimand')
- 6 State assumptions validating the causal effect estimation from the available data.
- 7 Estimate target causal effect.
- 8 Evaluate the validity of the assumptions and perform sensitivity analyses as needed.

## The Simulation Learner

- Simulate dataset based on an existing study
- Augment data with potential outcomes and potential exposures
- Aim: Illustrates concepts and methods on data



## Estimands for time to event outcome

Needed:

- Relevant time-to-event  $\tilde{T}$  as outcome variable;
- A clear starting point (time zero);
- Population of interest (whole/ marginal, subpopulation/ conditional, the treated);
- (point) exposure/treatment of interest with interventions to compare: e.g. treat versus not-treat
- Potential outcomes:  $\tilde{T}^a$  is potential time-to-event under treatment setting  $A = a$ .

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Concept of emulating Target Trials can be very helpful  
(Hernán(2016))

## Estimands: censoring

- Censoring makes the event of interest 'invisible';
- Interest in inference for an uncensored population;
- Estimand should not depend on censoring aspects;

A special type of intermediate event

## Estimands: defining time zero

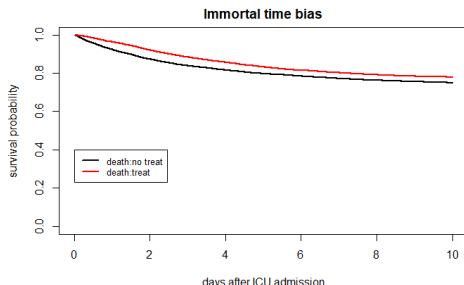
- In RCTs: time zero is clear (start of randomisation)
- In observational studies often less so
- Different time scales: calendar time versus start treatment versus age
- Prevalent instead of incident users  $\Rightarrow$  selection bias
- Users defined later in follow up  $\Rightarrow$  immortal time bias

## Immortal time bias: a common made mistake

Simulation example: based on recent COVID research

- Population: patients admitted to ICU
- A treatment without an effect
- Treatment started between day 0 and 5

Ignoring delayed start of treatment yields:



Observed effect (HR=0.88), while no effect was present

## The COVID example: issues when defining estimand

- What would be the population of interest? All patients at the ICU? The treated subpopulation? Or those "admissible for treatment"?
- What are the levels of treatment? "Start immediately" versus "no treatment"? Or some dynamic treatment strategy( e.g start treatment if oxygen levels are very low)

## Estimands: the scale of the contrast

### 1. Survival/risk scale

- Differences/ratios in survival probabilities at specific time point(s)  $P(\tilde{T}^1 > t)$  versus  $P(\tilde{T}^0 > t)$ ;
- Difference in survival curves;
- Difference in median survival time;
- Difference in (restricted) mean survival time  $E(\min(\tilde{T}^1, t^*))$  versus  $E(\min(\tilde{T}^0, t^*))$ , with  $t^*$  a predefined time horizon

## Estimands: hazard scale

Hazard ratios are extremely popular in survival analysis

- They can be estimated (assuming non informative censoring) without specifying baseline hazard
- Theory and software of Cox proportional hazard model are well developed
- Often low dimensional models fit well

But

- Hazard ratios are non collapsible (conditional hazards differ from marginal hazards)
- Hazards and hazard ratios have difficult causal interpretation (hazard of hazard ratios (Hernán (2010); Aalen et al(2015); Stensrud et al(2019); Martinussen et al (2020), Young et al (2020))



## A causal view on hazards

Causal hazard function:

$$\lambda^a(t) = \lim_{h \rightarrow 0} \frac{1}{h} P(t \leq \tilde{T}^a < t + h \mid \tilde{T}^a \geq t).$$

- Interest in contrast between  $\lambda^1(t)$  and  $\lambda^0(t)$ .
- When patients have different frailties: populations with  $\tilde{T}^1 \geq t$  and  $\tilde{T}^0 \geq t$  after some time no longer exchangeable
- Relatively more frail people at risk in treated group.
- Conditioning  $\Rightarrow$  Selection bias
- Counter-intuitive things can happen:
  - $\tilde{T}_i^1 > \tilde{T}_i^0$  for all individuals  $i$
  - while  $\lambda^1(t) \leq \lambda^0(t)$  for some  $t$ .
- Even if  $A$  is randomised.

## Estimands on 'speed' scale

Accelerated failure times

$$T_i^*(1) = T_i^*(0) \exp(\psi_i).$$

or

$$\log(T_i^*(1)) = \log(T_i^*(0)) + \psi_i.$$

An estimand in the population could be

$$\psi = E(\log T^*(1)) - E(\log T^*(0))$$

## General assumptions needed for estimation

- Positivity; each individual should be able to receive all treatment levels
- Causal consistency: observed outcome is equal to an potential outcome when  $A$  is set to observed treatment level
- No interference: impact of treatment on outcome is not affected by other individuals being exposed or not.

## Other assumptions needed for estimation

- No unmeasured confounding (NUC); received treatment is independent of the potential outcomes, given baseline covariates.
- Or alternative sets of assumptions (e.g. instrumental variable methods, discontinuity designs )

## Methods based on NUC: 1. Outcome regression with standardisation

- Predict survival curves for each individual, given all (measured) confounders, separately for treated and untreated level
- Then average over empirical covariate distribution, separate for treated and untreated
- Direct standardisation
- Yields average survival curves in total population
- Easy R package `stdReg` (Sjolander (2016)) or `stpm2` in Stata

## Methods based on NUC: 2.Propensity score methods

- Fit a model for the propensity of receiving treatment conditional on the covariates,  $e(L) = P(A = 1|L)$
- Check if fitted PS is adequate (well balanced)
- Then use PS methods (e.g matching, stratification, inverse probability weighting)

## Different underlying assumptions

- Outcome regression:
  - Correct formulation of survival model
  - Non informative censoring, conditional on covariates in model
- Propensity score methods:
  - Correct propensity model
  - Non informative censoring

## Example: pre-emptive kidney transplant and survival

Generate a simulation setting, motivated by real data example

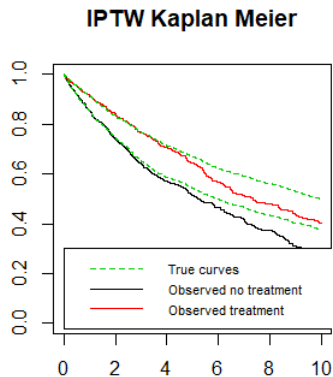
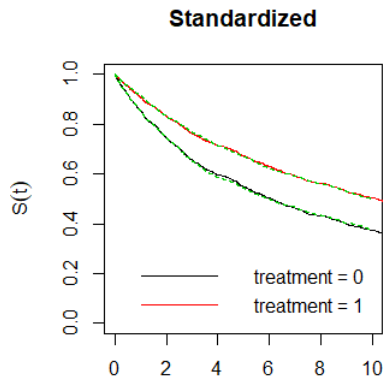
- Patients become eligible for kidney transplant
- Treatment: transplanted immediately versus later
- Generated 2000 patients, included between 2001 and 2017
- Treatment depends on age and calendar year
- Survival (exponential) depends on treatment, age, interaction between age and treatment and year
- Over time mean age in population increases
- Administrative censoring in 2020



## Estimation

- Outcome regression, with standardisation
- Propensity with inverse probability weighting
- In both models age and year as covariates

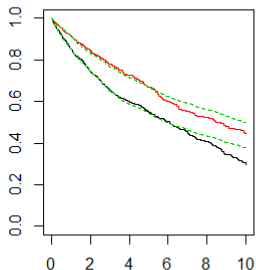
## Dealing with informative censoring



Propensity methods need to account for informative censoring

## Using IPTW + censoring weights

**IPTW and censoring weights**



- Censoring model I used, is probably not perfect
- If censoring depends on time varying covariates, both methods need censoring weights

## Competing events

- Competing event  $Z$ , with time to occurrence  $V$
- Here  $Z$  can affect  $\tilde{T}^a$ , as the event of interest will never happen after  $Z$  occurs.
- Different estimands may be of interest
  - ① contrasts between  $\tilde{T}^1$  versus  $\tilde{T}^0$ 
    - the 'total' effect of  $A$  on  $\tilde{T}$ .
    - corresponds to "competing risk cumulative incidences".
  - ② contrasts between  $\tilde{T}^{1, V=\infty}$  versus  $\tilde{T}^{0, V=\infty}$ 
    - 'Direct effect' effect of  $A$  on  $\tilde{T}^a$ .
    - What would happen if we eliminate  $Z$ ?

Estimand 2 is what typically be used by many as default, without being aware of it.

## Conclusion

Think careful about estimand of interest, and about corresponding analysis with underlying assumptions

[www.ofcaus.org](http://www.ofcaus.org)



## References

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- Aalen et al. Lifetime Data Anal 2015
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